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Spectral flow on the Higgs branch and AdS/CFT duality

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Abstract: We use AdS/CFT duality to study the large N_c limit of the meson spectrum on the Higgs branch of a strongly coupled, $\mathcal{N} = 2$ supersymmetric $SU(N_c)$ gauge theory with $N_f = 2$ fundamental hypermultiplets. In the dual supergravity description, the Higgs branch is described by SU(2) instanton configurations on D7-branes in an AdS background. We compute the spectral flow parameterized by the size of a single instanton. In the large N_c limit, there is a sense in which the flow from zero to infinite instanton size, or Higgs VEV, can be viewed as a closed loop. We show that this flow leads to a non-trivial rearrangement of the spectrum.

Keywords: Nonperturbative Effects, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

The spectrum of strongly coupled large N_c gauge theories can in many cases be computed using the holographically dual description of the AdS/CFT correspondence [1 − 3]. In its original form, this duality was applicable to gauge theories with adjoint fields only. In examples of the duality for confining gauge theories, the spectrum of glueballs can be determined from the spectrum of normalizable classical solutions describing small fluctuations about a dual supergravity background [4]. Explicit computations appear in [5 − 8].

Much effort has gone into extending AdS/CFT duality to include theories with fundamental representations. The earliest example of AdS/CFT duality for a theory with fundamental representations related a conformal $\mathcal{N} = 2$ Sp(N) gauge theory to string theory in $AdS_5 \times S^5/\mathbb{Z}_2$, with D7-branes wrapping the $\mathbb{Z}_2$ fixed surface with geometry $AdS_5 \times S^3$ [9, 10](see also [11] for related early work). In [12], this duality was extended to an $\mathcal{N} = 2$ SU(N_c) theory with N_f massive fundamental hypermultiplets, essentially by removing the $\mathbb{Z}_2$ orientifold, which was justified by the fact that a probe D7-brane wrapping a contractible S^3 does not lead to a tadpole requiring cancellation. The dual field theory is not asymptotically free, but has a UV fixed point in the strict $N_c \to \infty$ limit. Subsequent to this, there have also been a number of papers generalizing the duality to confining theories with fundamental representations [13 − 21], including non-supersymmetric examples in which chiral symmetry breaking by a $\bar{\Psi}\Psi$ quark condensate occurs [22 − 31].

In this paper, we revisit the $\mathcal{N}=2$ theory at large N_c whose AdS description was constructed in [12]. The meson spectrum of this theory at the origin of moduli space was computed exactly in [32] by solving the Dirac-Born-Infeld equations of motion for small fluctuations of the D7-brane embedded in the AdS background. Other properties of this theory were discussed in [33, 34]. We extend the analysis of [32] to include the spectrum at points on the mixed Coulomb-Higgs branch. The AdS description of these points in moduli space has been discussed in [35−37], and involves gauge field backgrounds on the embedded D7-branes with non-zero instanton number. We explicitly consider the part of the Higgs branch dual to a single instanton, and compute the spectrum as a function of the instanton size. There is a sense in which the zero size and infinite size limits are equivalent, modulo a singular gauge transformation. In the dual large N_c gauge theory, this is an equivalence between the spectrum of the $\mathrm{SU}(N_c)$ theory and the $\mathrm{SU}(N_c-1)$ theory obtained by taking the Higgs VEV to infinity. We shall see that the spectral flow between these limits leads to a non-trivial re-arrangement of the mass eigenstates and global charges. Since our purpose is to exhibit this flow, we shall focus on a particular meson vector multiplet with small global symmetry charges, rather than computing the entire spectrum accessible to supergravity methods. The specific flow we consider takes vector mesons in the $(0,0)$ representation of a global $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ symmetry, which is unbroken at the origin of moduli space, to vector mesons in the representation $(1,1)$.

The organization of this paper is as follows. In section 2, we review the AdS description of the Higgs branch of the $\mathcal{N}=2$ theory with fundamental hypermultiplets described in [12]. Section 3 contains a review of the AdS description of the Higgs branch and parts of the mixed Coulomb-Higgs branch. In section 4, we discuss the AdS/CFT dictionary at the points on the moduli space dual to a single instanton. In section 5, we compute the meson spectrum by solving for classical fluctuations about the instanton background.

2. SUGRA dual of an $\mathcal{N}=2$ theory with fundamental representations

The specific $\mathcal{N}=2$ gauge theory which we consider is dual to string theory in $AdS_5\times S^5$ with N_f D7-branes wrapping a surface which is asymptotically $AdS_5\times S^3$. This particular duality was originally described in [12]. The matter content of this gauge theory is that of the $\mathcal{N}=4$ $\mathrm{SU}(N_c)$ gauge theory, with an added N_f (possibly massive) fundamental hypermultiplets. In $\mathcal{N}=1$ superspace, the lagrangian is

$$\mathcal{L}=\mathrm{Im}\bigg[\tau\int d^2\theta d^2\bar\theta\left(\mathrm{tr}(\bar\Phi_I e^V\Phi_I e^{-V})+Q_i^\dagger e^V Q^i+\tilde Q_i e^V\tilde Q^{i\dagger}\right)+\\ +\tau\int d^2\theta\left(\mathrm{tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)+W\right)+\tau\int d^2\bar\theta\left(\mathrm{tr}(\bar{\mathcal{W}}_{\dot\alpha}\bar{\mathcal{W}}^{\dot\alpha})+\bar W\right)\bigg] \tag{2.1}$$

where the superpotential W is

$$W=\mathrm{tr}(\epsilon_{IJK}\Phi_I\Phi_J\Phi_K)+\tilde Q_i(m+\Phi_3)Q^i\,. \tag{2.2}$$

The superfields Q^i and $\tilde Q_i$, labeled by the flavor index $i=1,\ldots,N_f$, make up the $\mathcal{N}=2$ fundamental hypermultiplets.

Coordinates									
0	1	2	3	4	5	6	7	8	9
D3									
D7									
$x^{\mu,\nu,\dots}$				$y^{m,n,\dots}$				$z^{i,j,\dots}$	
				r					
				y					

Table 1: Index conventions.

At finite N_c this theory is not asymptotically free, and the corresponding string background suffers from an uncancelled tadpole. However, as in [12], we focus strictly on the $N_c \to \infty$ limit with fixed N_f. In this case the beta function for the 't Hooft coupling vanishes, and the dual AdS string background has no tadpole problem because the probe D7-branes wrap a contractible S^3. Although contractible, the background is stable. The tachyonic mode associated with shrinking the S^3 satisfies (saturates) the Breitenlohner-Freedman bound [38]. Furthermore the $AdS_5 \times S^3$ embedding has been shown to be supersymmetric [39].

The $AdS_5 \times S^5$ background is given by

$$\begin{aligned}
ds^2 &= H^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)(d\vec{y}^{\,2} + d\vec{z}^{\,2})\,, \\
H(r) &= \frac{L^4}{r^4}\,, & r^2 &= \vec{y}^{\,2} + \vec{z}^{\,2}\,, \\
L^4 &= 4\pi g_s N_c (\alpha')^2\,, & \vec{y}^{\,2} &= \textstyle\sum_{m=4}^{7} y^m y^m\,, \\
C^{(4)}_{0123} &= H^{-1}\,, & \vec{z}^{\,2} &= (z^8)^2 + (z^9)^2\,, \\
e^\phi &= e^{\phi_\infty} = g_s\,.
\end{aligned} \tag{2.3}$$

We have only written the components of the Ramond-Ramond four-form which will be relevant to our computations. N_f D7-branes are embedded in this geometry according to

$$z^8 = 0\,, \qquad z^9 = (2\pi\alpha')m\,. \tag{2.4}$$

This leads to the induced metric

$$ds^2_{\rm D7} = H^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)d\vec{y}^{\,2}\,, \tag{2.5}$$

$$r^2 = y^2 + (2\pi\alpha')^2 m^2\,, \qquad y^2 \equiv y^m y^m\,. \tag{2.6}$$

The parameter m corresponds to the mass of the fundamental hypermultiplets in the dual $\mathcal{N} = 2$ theory. For $m = 0$, the geometry (2.5) is $AdS_5 \times S^3$, while for $m \neq 0$, the geometry approaches $AdS_5 \times S^3$ at large r. As long as N_f is held fixed in limit $N_c \to \infty$ with fixed $\lambda = g_s N_c \gg 1$, there is no need to consider the back-reaction of the D7-branes on the bulk geometry.

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3. The Higgs branch

3.1 Field theory

The $\mathcal{N}=2$ theory to which the supergravity background (2.3)–(2.5) is dual contains an $\mathcal{N}=2$ vector multiplet, one adjoint hypermultiplet, and N_f fundamental hypermultiplets. In $\mathcal{N}=1$ language, the superpotential is

$$W = \tilde{Q}_i(m+\Phi_3)Q^i + \mathrm{tr}[\Phi_1,\Phi_2]\Phi_3\,. \tag{3.1}$$

The chiral superfields Q^i and $\tilde{Q}_i$ belong to the fundamental hypermultiplets, with flavor index i. Φ_1 and Φ_2 belong to an adjoint hypermultiplet, while Φ_3 belongs to the vector multiplet. We denote the (scalar) bottom components of the superfields by lowercase letters. On the Higgs branch, the vector multiplet moduli ϕ_3 vanish while q^i and $\tilde{q}_i$ have non-zero expectation values.[1] There are also mixed Coulomb-Higgs vacua, for which both $q^i, \tilde{q}_i$ and ϕ_3 have non-zero expectation values.

For non-zero m and vanishing ϕ_3, the fundamental hypermultiplets are massive and there is no Higgs branch. However there is a mixed Coulomb-Higgs branch when ϕ_3 has an expectation value such that some components of the hypermultiplets are massless. An example of a point on a mixed Coulomb-Higgs branch is given by a diagonal ϕ_3 for which all but the last k entries are vanishing:

$$\phi_3 = \begin{pmatrix} 0 & & & & & \\ & \ddots & & & & \\ & & 0 & & & \\ & & & -m & & \\ & & & & \ddots & \\ & & & & & -m \end{pmatrix}. \tag{3.2}$$

In this case, the F-flatness equations $\tilde{q}_i(\phi_3+m) = (\phi_3+m)q^i = 0$ permit fundamental hypermultiplet expectation values in which only the last k entries of q^i and $\tilde{q}_i$ are non-zero;

$$q^i = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \alpha_1^i \\ \vdots \\ \alpha_k^i \end{pmatrix}, \qquad \tilde{q}_i = \begin{pmatrix} 0 & \cdots & 0 & \beta_{1i} & \cdots & \beta_{ki} \end{pmatrix}. \tag{3.3}$$

There are additional F and D-flatness constraints which we have not explicitly written.

In string theory, nonzero entries in (3.3) physically correspond to D3-branes which are coincident with and dissolved within the D7-branes. Dissolved D3-branes can be viewed as instantons in the eight-dimensional world-volume theory on the D7-branes [40], due to

[1] The scalars ϕ_1 and ϕ_2 belonging to the adjoint hypermultiplet may also have non-zero expectation values on the Higgs branch.

the Wess-Zumino coupling

$$S_{\rm WZ} = \frac{T_7(2\pi\alpha')^2}{4}\int C|_{PB}^{(4)} \wedge {\rm tr}(F\wedge F)\,. \tag{3.4}$$

In fact, there is a known exact map between the moduli space of Yang-Mills instantons and the Higgs branch of the $p+1$ dimensional theory arising on the Dp — D$p+4$ intersections. The ADHM constraints from which instantons are constructed are equivalent to the F and D-flatness equations of the $p+1$ dimensional theory [41, 42] (see also [43] for a review). The existence of instanton solutions for the D7-brane embedded in (2.3) according to (2.4) is a non-trivial consequence of AdS/CFT duality [35 – 37].

3.2 Supergravity description of the Higgs branch

The AdS/CFT dictionary relates the fundamental hypermultiplets of the $\mathcal{N}=2$ theory to degrees of freedom on D7-branes embedded in the AdS geometry according to (2.4). In light of the one-one correspondence between instantons and the Higgs branch, one expects that the supergravity description of the Higgs branch involves instanton solutions of the non-abelian Dirac-Born-Infeld action which describes the D7 branes.

The effective action describing D7-branes in a curved background is

$$\begin{aligned} S = {} & T_7 \int \sum_r C^{(r)} \wedge {\rm tr}\, e^{2\pi\alpha' F} + \\ & + T_7 \int d^8\xi\, \sqrt{g} e^{-\phi} (2\pi\alpha')^2 \frac{1}{2}\,{\rm tr}\left(F_{\alpha\beta}F^{\alpha\beta}\right) + \cdots\,, \end{aligned} \tag{3.5}$$

where we have not written terms involving fermions and scalars. This action is the sum of a Wess-Zumino term, a Yang-Mills term, and an infinite number of corrections at higher orders in α' indicated by $\cdots$ in (3.5). Since we need to consider at least two flavors (two D7's) in order to have a Higgs branch, the DBI action is non-abelian. The correspondence between instantons and the Higgs branch suggests that the equations of motion should be solved by field strengths which are self-dual with respect to a flat four-dimensional metric. In this paper, we work to leading order only in the large 't Hooft coupling expansion generated by AdS/CFT duality, which allows one to only consider the leading term in the α' expansion of the action. Constraints on unknown higher order terms arising from the existence of instanton solutions, as well as the exactly known metric on the Higgs branch, were discussed in [35, 37].

At leading order in α', field strengths which are self dual with respect to the flat four dimensional metric $ds^2 = \sum_{m=1}^4 dy^m dy^m$ solve the equations of motion, due to a conspiracy between the Wess-Zumino and Yang-Mills term. Inserting the explicit AdS background values (2.3) for the metric and Ramond-Ramond four-form into the action for D7-branes embedded according to (2.4), with non-trivial field strengths only in the directions y^m, gives

$$\begin{aligned} S &= \frac{T_7(2\pi\alpha')^2}{4}\int d^4x\, d^4y\, H(r)^{-1}\left(-\frac{1}{2}\epsilon_{mnrs}F_{mn}F_{rs} + F_{mn}F_{mn}\right) \\ &= \frac{T_7(2\pi\alpha')^2}{2}\int d^4x\, d^4y\, H(r)^{-1} F_-^2\,, \end{aligned} \tag{3.6}$$

where $r^2 = y^m y^m + (2\pi\alpha' m)^2$ and $F^-_{mn} = \frac{1}{2}(F_{mn} - \frac{1}{2}\epsilon_{mnrs}F_{rs})$. Field strengths $F^-_{mn} = 0$, which are self-dual with respect to the flat metric $dy^m dy^m$, manifestly solve the equations of motion. These solutions correspond to points on the Higgs branch of the dual $\mathcal{N} = 2$ theory. Strictly speaking, this is a point on the mixed Coulomb-Higgs branch if $m \neq 0$, with expectation values of the form (3.2), (3.3). We emphasize that in order to neglect the back-reaction due to dissolved D3-branes, we are considering a portion of the moduli space for which the instanton number k is fixed in the large N_c limit.

4. Higgs branch AdS/CFT dictionary

For simplicity, we consider the case $N_f = 2$, corresponding to two D7-branes, which is the minimum value giving a non-trivial Higgs branch. For $m = 0$, the AdS geometry (2.3) together with the embedding (2.4), (2.5), is invariant under $\mathrm{SO}(2,4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_R \times \mathrm{SU}(2)_f$. The combination $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ acts as SO(4) rotations of the coordinates y^m. The SO(2,4) factor is the conformal symmetry of the dual gauge theory. The $\mathrm{SU}(2)_L$ factor corresponds to a global symmetry of the dual gauge theory, while $\mathrm{SU}(2)_R \times \mathrm{U}(1)_R$ corresponds to the R symmetries. Finally $\mathrm{SU}(2)_f$ is the gauge symmetry of the two coincident D7-branes which, at the AdS boundary, corresponds to the flavor symmetry of the dual gauge theory.

For $m \neq 0$, the symmetry is broken to $\mathrm{SO}(1,3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_f$. This is broken further if there is an instanton background on the D7-branes. We focus on that part of the Higgs branch, or mixed Coulomb-Higgs branch, which is dual to a single instanton centered at the origin $y^m = 0$. The instanton, in "singular gauge," is given by

$$A_\mu = 0, \qquad A_m = \frac{2\Lambda^2 \bar\sigma_{nm} y_n}{y^2(y^2+\Lambda^2)}\,, \tag{4.1}$$

where Λ is the instanton size, and

$$\begin{aligned} \bar\sigma_{mn} &\equiv \frac{1}{4}(\bar\sigma_m\sigma_n - \bar\sigma_n\sigma_m)\,, \qquad & \sigma_m &\equiv (i\vec\tau, 1_{2\times2})\,, \\ \sigma_{mn} &\equiv \frac{1}{4}(\sigma_m\bar\sigma_n - \sigma_n\bar\sigma_m)\,, \qquad & \bar\sigma_m &\equiv \sigma_m^\dagger = (-i\vec\tau, 1_{2\times2})\,. \end{aligned} \tag{4.2}$$

with $\vec\tau$ being the three Pauli-matrices. We choose singular gauge, as opposed to the regular gauge in which $A_n = 2\sigma_{mn}y^m/(y^2+\Lambda^2)$, because of the improved asymptotic behavior at large y. In the AdS setting, the Higgs branch should correspond to a normalizable deformation of the background at the origin of the moduli space. The singularity of (4.1) at $y^m = 0$ is not problematic for computations of physical (gauge invariant) quantities.

The instanton (4.1) breaks the symmetries to

$$G = \mathrm{SO}(1,3) \times \mathrm{SU}(2)_L \times \mathrm{diag}(\mathrm{SU}(2)_R \times \mathrm{SU}(2)_f)\,, \tag{4.3}$$

and corresponds to a point on the Higgs branch

$$q_{i\alpha} = v\,\varepsilon_{i\alpha}\,, \qquad v = \frac{\Lambda}{2\pi\alpha'}\,, \tag{4.4}$$

where $q_{i\alpha}$ are scalar components of the fundamental hypermultiplets, labeled by a $\mathrm{SU}(2)_f$ index $i = 1, 2$, and a $\mathrm{SU}(2)_R$ index $\alpha = 1, 2$. All the broken symmetries are restored in the ultraviolet (large r), where the theory becomes conformal.

5. Fluctuation spectrum

To determine the spectrum on the Higgs branch, we now consider fluctuations about the instanton background (4.1). There are some obvious fluctuations of A_m dual to massless scalar mesons, namely fluctuations corresponding to changes in the instanton moduli. We focus instead on fluctuations of A_μ in the lowest representations of the un-broken $\mathrm{SU}(2)_L \times \mathrm{diag}(\mathrm{SU}(2)_R \times \mathrm{SU}(2)_f)$ which are dual to vector mesons, as well as the scalar fluctuations belonging to the same super-multiplet.

In terms of coordinates x^μ, y^m on the D7-brane world-volume, with the former corresponding to the space-time directions of the dual gauge theory, the D7-brane action is

$$\begin{aligned} S_{\mathrm{D7}} = \frac{(2\pi\alpha')^2 T_7}{4} \int d^4x\, d^4y \ \mathrm{tr}\Big[& H(r) F_{\mu\nu} F_{\mu\nu} + 2 F_{m\nu} F_{m\nu} + \\ & + H^{-1}(r)\, \frac{1}{2} (F_{mn} - {}^*F_{mn})(F_{mn} - {}^*F_{mn}) \Big] , \end{aligned} \tag{5.1}$$

with $r^2 = y^m y^m + (2\pi\alpha' m)^2$. We have excluded scalar and fermionic terms, as well as higher dimension operators which give rise to subleading corrections in the large 't Hooft coupling expansion. The equations of motion for this action are

$$H(r)\mathcal{D}_\mu F_{\mu\nu} + \mathcal{D}_m F_{m\nu} = 0 \,, \tag{5.2a}$$

$$\mathcal{D}_\mu F_{\mu n} + \mathcal{D}_m \left[H^{-1}(r)(F_{mn} - {}^*F_{mn}) \right] = 0 \,. \tag{5.2b}$$

Let us consider the equations of motion for small fluctuation $\mathcal{A}$ about the instanton, defined by $\mathcal{A} \equiv A - A^{\mathrm{inst}}$. The normalizable solutions of the classical equations for fluctuations $\mathcal{A}_\mu$ with $\mathcal{A}_m = 0$ determine the spectrum of vector mesons. To linear order in $\mathcal{A}_\mu$, the equations of motion are

$$\mathcal{D}_n(\partial_\mu \mathcal{A}_\mu) = 0 \,, \tag{5.3a}$$

$$\begin{aligned} H\partial_\mu\partial_\mu\mathcal{A}_\nu + \partial_m\partial_m\mathcal{A}_\nu + \partial_m \left[A_m^{\mathrm{inst}}, \mathcal{A}_\nu \right] + & \\ + \left[A_m^{\mathrm{inst}}, \partial_m \mathcal{A}_\nu \right] + \left[A_m^{\mathrm{inst}}, \left[A_m^{\mathrm{inst}}, \mathcal{A}_\nu \right] \right] &= 0 \,. \end{aligned} \tag{5.3b}$$

The simplest (non-abelian) ansatz for the fluctuations $\mathcal{A}_\mu$ is given by

$$\mathcal{A}_\mu{}^{(a)} = \xi_\mu(k) f(y) e^{ik_\mu x_\mu} \tau^a \,, \qquad y^2 \equiv y^m y^m \,, \tag{5.4}$$

which is a singlet under $\mathrm{SU}(2)_L$ and a triplet under $\mathrm{diag}(\mathrm{SU}(2)_R \times \mathrm{SU}(2)_f)$. Equation (5.3a) is solved by $k_\mu \xi_\mu = 0$, while (5.3b) becomes

$$0 = \left[\frac{M^2 L^4}{(y^2 + (2\pi\alpha')^2 m^2)^2} - \frac{8\Lambda^4}{y^2(y^2+\Lambda^2)^2} + \frac{1}{y^3}\partial_y(y^3\partial_y) \right] f(y) \,, \tag{5.5}$$

where $M^2 = -k_\mu k_\mu$. To determine the spectrum, we must find the values of M^2 for which this equation admits normalizable solutions.

It is convenient to work in units of the hypermultiplet mass by defining

$$\tilde{y} \equiv \frac{y}{2\pi\alpha' m}\,, \qquad \tilde{\Lambda} \equiv \frac{\Lambda}{2\pi\alpha' m}\,, \qquad \tilde{M}^2 \equiv \frac{M^2 L^4}{(2\pi\alpha' m)^2}\,, \tag{5.6}$$

such that equation (5.5) becomes

$$0 = \left[\frac{\tilde{M}^2}{(\tilde{y}^2+1)^2} - \frac{8\tilde{\Lambda}^4}{\tilde{y}^2(\tilde{y}^2+\tilde{\Lambda}^2)^2} + \frac{1}{\tilde{y}^3}\partial_{\tilde{y}}(\tilde{y}^3\partial_{\tilde{y}})\right] f(\tilde{y})\,. \tag{5.7}$$

This equation can be solved analytically in the limits of zero or infinite instanton size. For finite $\tilde{\Lambda}$ we solve it numerically.

At large $\tilde{y}$, (5.7) has two linear independent solutions whose asymptotics are given by $\tilde{y}^{-\lambda}$ with $\lambda = 0, 2$. The normalizable solutions corresponding to vector meson states behave as $\tilde{y}^{-2}$ asymptotically. From the standard AdS/CFT correspondence, one expects $\lambda = \Delta$ and $\lambda = 4 - \Delta$, where Δ is the UV conformal dimension of the lowest dimension operator with the quantum numbers of the vector meson. However, the kinetic term does not have a standard normalization, i.e. the radial component of the Laplace operator appearing in the equation above is not (only) $\partial^2_{\tilde{y}}$, and consequently an extra factor of $\tilde{y}^\alpha$, for some α, appears in the expected behaviour; so we have $\lambda = \alpha + \Delta$, $\alpha + 4 - \Delta$. From the difference we conclude that $\Delta = 3$. The dimensions and quantum numbers are those of the SU(2) flavor current,

$$\mathcal{J}^b_\mu = -\bar{\psi}^{\pm i}\gamma_\mu \sigma^b{}_{ij}\psi_\mp{}^j + \bar{q}^{\alpha i}\overleftrightarrow{D}_\mu\, \sigma^b{}_{ij}\, q_\alpha{}^j\,, \tag{5.8}$$

with α the $\mathrm{SU}(2)_R$ index and i, j the flavor indices. This current has $\mathrm{SU}(2)_R \times \mathrm{SU}(2)_L \times \mathrm{U}(1)$ quantum numbers $(0,0)_0$.

The asymptotic behavior of the supergravity solution is

$$\mathcal{A}^\mu_{b(a)} = \xi^\mu(k) e^{ik_\mu x^\mu} f(\tilde{y})\delta_{ab} \sim \tilde{y}^{-2}\langle a, \xi, k|\mathcal{J}^\mu_b(x)|0\rangle\,, \tag{5.9}$$

where $\mathcal{J}^\mu$ is the SU(2) flavor current and $|a, \xi, k\rangle$ is a vector meson with polarization ξ, momentum k, and flavor triplet label a. Note that the index b in $\mathcal{A}^\mu_{b(a)}$ is a Lie algebra index, whereas the index (a) labels the flavor triplet of solutions.

For small radii $\tilde{y}$, the asymptotics of the general solution of (5.7) is $c_1\tilde{y}^2 + c_2\tilde{y}^{-4}$. Requiring a normalizable solution with regular behavior at the origin gives a discrete spectrum of allowed $\tilde{M}^2$. We have calculated $\tilde{M}^2$ for the lowest-lying modes using a standard numerical shooting method. The results are shown in figure 1.

Because of $\mathcal{N} = 2$ supersymmetry, there will be a number of other fluctuations with the same spectrum as the vector mesons, which are required to fill out massive $\mathcal{N} = 2$ vector multiplets. These include, for example, fluctuations of the adjoint scalars on the D7-brane. At quadratic order, the part of the D7-brane action involving these scalars and the corresponding equation of motion is

$$\begin{aligned} &S_{D7} = -\tfrac{(2\pi\alpha')^2 T_7}{4}\int d^4x\, d^4y\ \mathrm{tr}\left[H(r)\mathcal{D}_\mu\Phi\mathcal{D}_\mu\bar{\Phi} + \mathcal{D}_m\Phi\mathcal{D}_m\bar{\Phi}\right], \\ &H(r)\partial_\mu\partial_\mu\Phi + \mathcal{D}_m\mathcal{D}_m\Phi = 0\,, \end{aligned} \tag{5.10}$$

where

$$\mathcal{D}_m\mathcal{D}_m\Phi = \partial_m\partial_m\phi + \left[A_m^{\rm inst},\, \partial_m\Phi\right] + \partial_m\left[A_m^{\rm inst},\, \Phi\right] + \\ + \left[A_m^{\rm inst},\, \left[A_m^{\rm inst},\, \Phi\right]\right]\,. \tag{5.11}$$

As (5.10) coincides with the equation of motion for the gauge field (5.3b), except for the vector index present, the ansatz for the scalar field

$$\Phi = f(\tilde{y})e^{ik_\mu x_\mu}\tau^a \tag{5.12}$$

yields exactly the same differential equation (5.5).

The scalar fluctuations (5.12) are dual to the descendant $QQ(q_i\bar{q}^i)$ of the scalar bilinear $q_i\bar{q}^i$, which has conformal dimension $\Delta = 3$. At $\Lambda = 0$ the scalar bilinear is in the $(0,0)$ representation of the unbroken $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry.

5.1 Spectral flow

In the limits of zero or infinite instanton size, the equations (5.7) become

$$0 = \left[\frac{\tilde{M}^2}{(\tilde{y}^2+1)^2} - \frac{l(l+2)}{\tilde{y}^2} + \frac{1}{\tilde{y}^3}\partial_{\tilde{y}}(\tilde{y}^3\partial_{\tilde{y}})\right] f(\tilde{y})\,, \tag{5.13}$$

with $l = 0, 2$ for zero or infinite $\tilde{\Lambda}$ respectively. Note that these are the same equations found in [32] for fluctuations about the trivial background $A_\mu = A_m = 0$ of the form

$$\mathcal{A}^\mu = \xi^\mu(k)e^{ik_\mu x^\mu} f(y)\mathcal{Y}_l(S^3)\,, \tag{5.14}$$

where $\mathcal{Y}_l(S^3)$ are spherical harmonics on S^3, which transform as $(\frac{l}{2}, \frac{l}{2})$ representations under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Regular, normalizable solutions exist for [32]

$$\tilde{M}^2 = 4(n+l+1)(n+l+2)\,, \tag{5.15}$$

for integer $n \geq 0$. The spectra computed in [32] are valid for a single flavor, for which there is no Higgs branch, or for multiple flavors at the origin of moduli space.

In the limit of infinite instanton size, the field strength vanishes locally, and one would again expect that the spectrum is the same as that at the origin of moduli space. In the dual gauge theory, the infinite size limit corresponds to the limit of infinite Higgs VEV, which reduces the gauge group from $\mathrm{SU}(N_c)$ to $\mathrm{SU}(N_c - 1)$. The distinction between $\mathrm{SU}(N_c)$ and $\mathrm{SU}(N_c - 1)$ is negligible in the large N_c limit. Thus, in some sense, a flow from zero to infinite size is a closed loop beginning and ending at the origin of moduli space. The spectrum is the same, although it could rearrange itself in a non-trivial way during the flow. This is indeed the case, as can be seen from figure 1*b*. We shall argue below that this flow takes vector mesons in the $(0,0)$ representation of the global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry, which is unbroken at the origin of the moduli space, to vector mesons in the representation $(1,1)$.

In singular gauge, the infinite size instanton is given by

$$A_n = 2\frac{\bar{\sigma}_{mn}y^m}{y^2} \tag{5.16}$$

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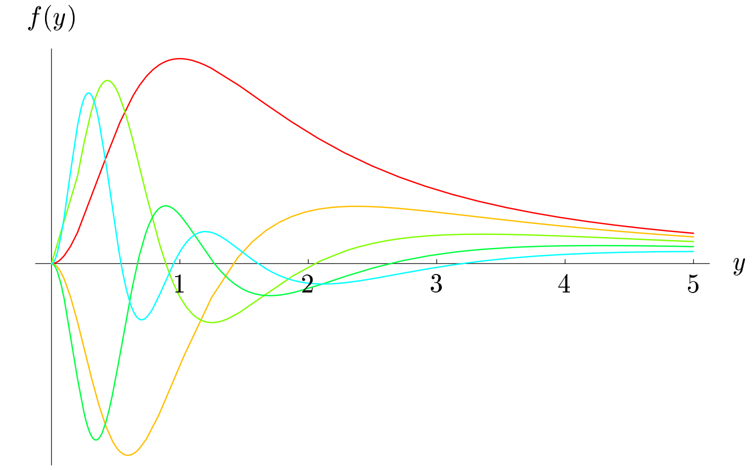
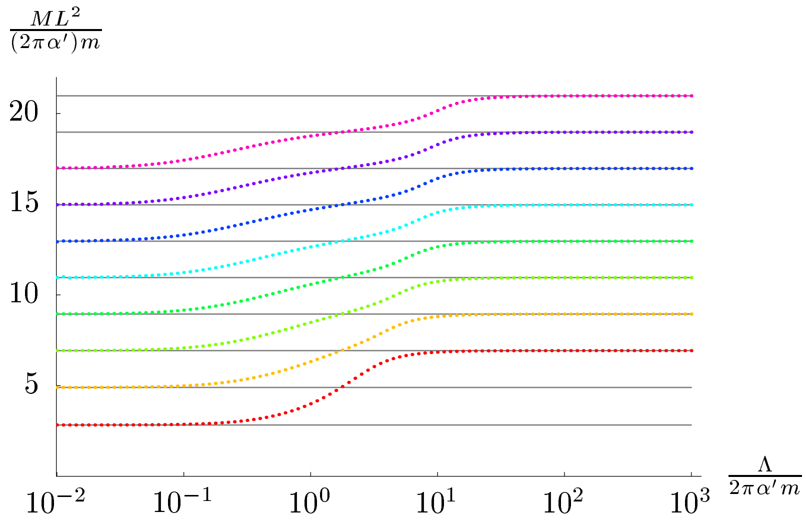


(a) Regular solutions of (5.7). The f axis has arbitrary scale.



(b) Numerically determined meson masses as function of the Higgs VEV.

Figure 1: Each dotted line represents a regular solution of the equation of motion, corresponding to a vector multiplet of mesons. The vertical axis in (b) is $\sqrt{\lambda}M/m$ where M is the meson mass, λ the 't Hooft coupling and m the quark mass. The horizontal axis is v/m where $v = \Lambda/2\pi\alpha'$ is the Higgs VEV. In the limits of zero and infinite instanton size (Higgs VEV), one recovers the spectrum (gray horizontal lines) obtained analytically in the absence of an instanton background by [32].

such that the ansatz (5.4) gives (5.13) with $l = 2$. Note however that the solution of the form (5.4) involves the trivial spherical harmonic (a constant) on S^3. Naively, it would seem impossible for a flow between zero and infinite instanton size to generate an $l = 2$ spherical harmonic on S^3, starting with the trivial $l = 0$ harmonic, since $\mathrm{SU}(2)_L$ is unbroken by the instanton (4.1). However, in the limit of infinite size, the correspondence with the the spectrum computed in [32] at the origin of moduli space is apparent only after making the

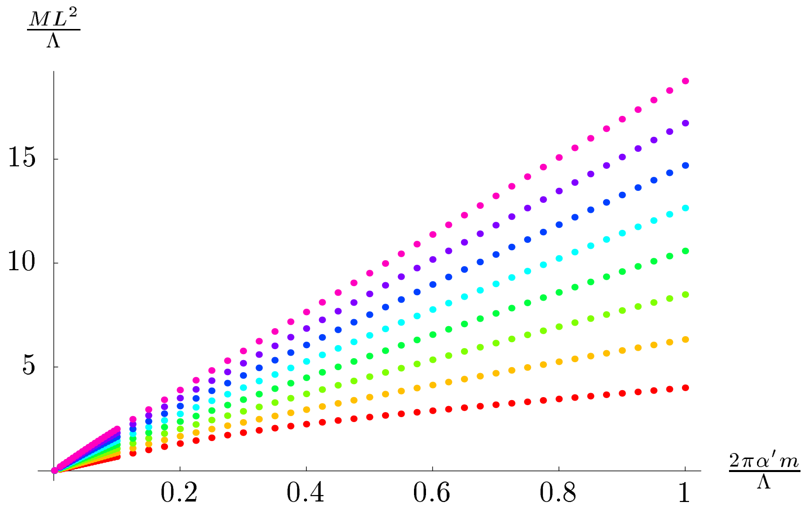


Figure 2: Numerical results for the meson mass spectrum as function of the quark mass. Both for $m/\Lambda \to 0$ and for $m/\Lambda \to \infty$, the curves become linear, however with different slope. The asymptotic slopes correspond to the constant values approached in figure 1(*b*).

gauge transformation

$$U = \frac{\sigma^m y^m}{|y|}, \tag{5.17}$$

which gives[2] $A_n = 0$. Acting on the ansatz (5.4), this gauge transformation gives

$$\mathcal{A}_\mu{}^{(a)} = \xi_\mu(k) f(y) e^{ik_\mu x_\mu} \hat{y}^m \hat{y}^n \sigma^m \tau^a \bar{\sigma}^n, \tag{5.18}$$

with $\hat{y}^m = y^m/|y|$. The matrix elements of $\mathcal{A}_\mu$ contain only $l = 2$ spherical harmonics on S^3, since $\hat{y}^n \hat{y}^m$ multiplies $C^{nm} \equiv \sigma^m \tau^a \bar{\sigma}^n$ which has zero trace: $C^{mm} = \sigma^m \tau^a \bar{\sigma}^m = 0$. Note that the singular gauge transformation (5.17) is large[3] and does not leave physical states or the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_f$ global charges invariant.

6. Conclusions

We have used AdS/CFT duality to compute meson spectra on the Higgs branch of a strongly coupled $\mathcal{N} = 2$ gauge theory at large N_c. The Higgs branch is dual to instanton configurations on D7-branes embedded in an AdS geometry. The meson spectrum is determined by finding the normalizable solutions describing small fluctuations about this background. We have focused on a particular portion of the mixed-Coulomb Higgs branch which is dual to a single instanton centered at the origin. Our intention has not been to compute the full spectrum accessible to supergravity calculations, but rather to illustrate a non-trivial spectral flow as one varies the size of the instanton between zero and infinity.

[2]The instanton number is non-zero, so we can only get vanishing $F_{mn} = 0$ by taking the instanton size to infinity at fixed y^m. The AdS wavefunctions associated with the spectrum we are discussing are localized in a region which is fixed as the instanton size goes to infinity.

[3]This gauge transformation relates the "singular gauge" to the "regular gauge" (see for instance [43]).

To this end we have focused on a particular ansatz for solutions dual to vector mesons with the smallest global symmetry charges. In the large N_c limit, there is a sense in which the flow from zero to infinite instanton size, or Higgs VEV, is a non-contractible closed loop. Although the spectrum at the endpoints is the same, the flow leads to a non-trivial rearrangement of global symmetry charges and mass eigenstates.

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